17. State and prove Uniqueness Theorem for analytic function.

18. Explain the characterization of Mobius maps in terms of their fixed points.

19. State and prove Borel Caratheodary Theorem.

20. State and prove Casorati – Weierstrass Theorem.

SHANMUGA INDUSTRIES & SCIENCE COLLEGE LIBRARY

NOVEMBER/DECEMBER 2024

GMA31/DMA31 — COMPLEX ANALYSIS I

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Write the polar form of C-R equations.

2. Define logarithm of complex function.

3. What is meant by Jordan domain?

4. Define continuous differentiable.

5. Write the Magnification property of Mobius maps.

6. What are the fixed points of $f(z) = z^{-1}$?

7. State Maximum modulus principle.

8. What is meant by convex hull?

9. Find the removable singularity of $f(z) = \dfrac{z}{e^z - 1} (z \neq 0)$.

10. State Riemann removable singularity Theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) State and prove Antiderivative Theorem.

Or

(b) Find the sum of the series $1+\sum_{n=2}^{\infty}\frac{(z-4)^n}{n}$.

12. (a) Evaluate $I=\int_{\gamma} xdz$ where γ is the boundary of the square $[0,1]\times[0,1]$ with C considered as R^2.

Or

(b) State and prove Taylor's Theorem.

13. (a) Prove that the inverse of Mobius transformation is also a Mobius Transformation.

Or

(b) Find the image of the circle $|z|=r\ (r\neq 1)$ under the Cayley mapping $w=f(z)=\frac{z-i}{z+i}$:

SAINTGITS INSTITUTES ARTS & SCIENCE COLLEGE LIBRARY

14. (a) State and prove Minimum modulus Theorem for analytic function.

Or

(b) State and prove Fundamental Theorem of Algebra.

15. (a) Let N be a deleted neighbourhood of z_0 such that f is analytic in N. If f has an isolated singularity at z_0, show that z_0 is a pole of order n if and only if there are positive constants C_1 and C_2 such that $C_2\leq\left|(z-z_0)^n f(z)\right|\leq C_1$ for some deleted neighbourhood N_0 of z_0 such that $N_0\subset N$.

Or

(b) Discuss the singularity of $f(z)=\frac{z-1-i}{z^2-(4+3i)z+(1+5i)}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $f(z)=u(x,y)+iv(x,y)$ is differentiable at z_0, prove that the C-R equations hold at $= x_0+iy_0$,: $if_x(z_0)=f_y(z_0)$: or equivalently $u_x(x_0,y_0)=v_y(x_0,y_0)$ and $u_y(x_0,y_0)=-v_x(x_0,y_0)$.